Analysis of Discontinuities in Dielectric Waveguides by Means of the Least Squares Boundary Residual Method

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***Abstract*—A novel approach to analyze the discontinuities in open-type transmission lines is proposed. The method used is the least squares boundary residual technique which has been applied previously to treat the boundary-value problems in closed-type transmission lines. As an example of application of our approach, the reflected, transmitted, and radiated waves caused by the transverse displacement at the junction of two dielectric slab waveguides are calculated.**

I. Introduction

DISCONTINUITY problems in optical transmission lines are of great interest from both the theoretical and the practical points of view. So far, the discontinuities at the junctions between optical fibers and/or optical integrated circuits have been investigated experimentally and analyzed theoretically by several authors [1]–[11]. However, most of the theoretical analyses reported previously have restricted limitations of practical application, since they have ignored the radiated or reflected wave, or both, under the assumption of slight discontinuity.

In this paper, the discontinuities in dielectric waveguides are analyzed using the least squares boundary residual method instead of conventional boundary conditions. As an example of application of the proposed approach, the reflected, transmitted, and radiated waves caused by the transverse displacement at the junction of two single-mode dielectric slab waveguides are calculated. The results are compared with those obtained by other methods, and the features of the least squares boundary residual method are clarified. The method used in the present paper has been applied previously to treat the boundary-value problems in closed-type waveguides [12], [13]. To the authors' knowledge, however, this method has not yet been used to analyze the open-type waveguides such as dielectric waveguides.

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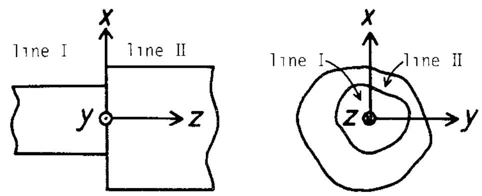

Fig. 1. Discontinuity at the junction of two dielectric waveguides. (a) Vertical section of the discontinuity between lines I and II. (b) Transverse cross section of the discontinuity at the junction of two lines.

II. Analysis of Discontinuity in Dielectric Waveguides

A. Least Squares Boundary Residual Method

The incoming wave incident upon the discontinuous junction between dielectric waveguides I and II as illustrated in Fig. 1 is reflected, transmitted, and radiated. Let $\boldsymbol{e}_{\mathrm{in}}$ and $\boldsymbol{h}_{\mathrm{in}}$ be the transverse components of the electric and magnetic fields of the incident wave, respectively, and $\boldsymbol{e}_i$, $\boldsymbol{h}_i$ and $\boldsymbol{e}(\rho), \boldsymbol{h}(\rho)$ be the electromagnetic fields of the ith guided mode and the radiation mode, respectively. ρ denotes the propagation constant of the radiation mode in the transverse direction outside the waveguide. The transverse components of the total electromagnetic fields $\boldsymbol{E}^{\mathrm{I}}$ and $\boldsymbol{H}^{\mathrm{I}}$ in line I and $\boldsymbol{E}^{\mathrm{II}}$ and $\boldsymbol{H}^{\mathrm{II}}$ in line II at the discontinuity plane ($z=0$) can be expressed in terms of the eigenmodes of line I and line II, respectively, as follows:

$$E^{\mathrm{I}} = e_{\mathrm{in}} + \sum_{i=1}^{M} R_i e_i^{\mathrm{I}} + \int R(\rho) \boldsymbol{e}^{\mathrm{I}}(\rho)\, d\rho$$

$$H^{\mathrm{I}} = h_{\mathrm{in}} - \sum_{i=1}^{M} R_i \boldsymbol{h}_i^{\mathrm{I}} - \int R(\rho) \boldsymbol{h}^{\mathrm{I}}(\rho)\, d\rho$$

$$E^{\mathrm{II}} = \sum_{i=1}^{N} T_i e_i^{\mathrm{II}} + \int T(\rho) \boldsymbol{e}^{\mathrm{II}}(\rho)\, d\rho$$

$$H^{\mathrm{II}} = \sum_{i=1}^{N} T_i \boldsymbol{h}_i^{\mathrm{II}} + \int T(\rho) \boldsymbol{h}^{\mathrm{II}}(\rho)\, d\rho \qquad (1)$$

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where R_i and T_i, $R(\rho)$ and $T(\rho)$ are the reflection and transmission coefficients of the ith guided mode and the radiation mode, respectively. M and N are the maximum numbers of the guided modes supported in lines I and II, respectively.

To transform the integral with respect to ρ in (1) into the discrete summations, let us expand $R(\rho)$ and $T(\rho)$ into the sum of proper functions in the form

$$R(\rho)=\sum r_i f_i(\rho)$$
$$T(\rho)=\sum t_i f_i(\rho). \qquad (2)$$

Substituting (2) into (1), we get

$$E^{\rm I}=e_{\rm in}+\sum_{i=1}^{M} R_i e_i^{\rm I}+\sum_{i=1}^{\infty} r_i E_i^{\rm I}$$
$$H^{\rm I}=h_{\rm in}-\sum_{i=1}^{M} R_i h_i^{\rm I}-\sum_{i=1}^{\infty} r_i H_i^{\rm I}$$
$$E^{\rm II}=\sum_{i=1}^{N} T_i e_i^{\rm II}+\sum_{i=1}^{\infty} t_i E_i^{\rm II}$$
$$H^{\rm II}=\sum_{i=1}^{N} T_i h_i^{\rm II}+\sum_{i=1}^{\infty} t_i H_i^{\rm II} \qquad (3)$$

where

$$E_i^{\rm I,II}=\int f_i(\rho)e^{\rm I,II}(\rho)\,d\rho$$
$$H_i^{\rm I,II}=\int f_i(\rho)h^{\rm I,II}(\rho)\,d\rho. \qquad (4)$$

Now, let us define the mean-square error of the transverse components of the electromagnetic fields between lines I and II at the discontinuity plane as follows:

$$F=\frac{\int_S\{|E^{\rm I}-E^{\rm II}|^2+Z_0^2|H^{\rm I}-H^{\rm II}|^2\}\,ds}{\int_S\{|e_{\rm in}|^2+Z_0^2|h_{\rm in}|^2\}\,ds} \qquad (5)$$

where Z_0 is an arbitrary impedance parameter and S is the discontinuity plane ($z=0$). In the following treatment, let us choose the value of Z_0 equal to the intrinsic impedance of the vacuum so that both the difference of electric field and magnetic field in (5) can be taken into account equally. F defined by (5) becomes zero for the exactly true values of the electromagnetic fields, since, from the boundary conditions, the transverse components of the electromagnetic fields $\boldsymbol{E}^{\rm I}$,$\boldsymbol{E}^{\rm II}$ and $\boldsymbol{H}^{\rm I}$,$\boldsymbol{H}^{\rm II}$ must be continuous across the discontinuity plane S provided that they are the correct fields. In other words, F represents the measure of the discrepancy from the satisfaction of the boundary conditions. In general, the transverse electromagnetic fields $\boldsymbol{E}^{\rm I,II}$ and $\boldsymbol{H}^{\rm I,II}$ consist of the true fields $\boldsymbol{E}_0^{\rm I,II}$,$\boldsymbol{H}_0^{\rm I,II}$ and the error fields $\delta\boldsymbol{E}^{\rm I,II}$,$\delta\boldsymbol{H}^{\rm I,II}$:

$$E^{\rm I,II}=E_0^{\rm I,II}+\delta E^{\rm I,II}$$
$$H^{\rm I,II}=H_0^{\rm I,II}+\delta H^{\rm I,II}. \qquad (6)$$

Substituting (6) into (5), the first variation of F can be derived as

$$\delta F=\frac{\int_S\left(\delta\boldsymbol{E}^{\rm I*}\cdot(\boldsymbol{E}_0^{\rm I}-\boldsymbol{E}_0^{\rm II})-\delta\boldsymbol{E}^{\rm II*}\cdot(\boldsymbol{E}_0^{\rm I}-\boldsymbol{E}_0^{\rm II})+Z_0^2\{\delta\boldsymbol{H}^{\rm I*}\cdot(\boldsymbol{H}_0^{\rm I}-\boldsymbol{H}_0^{\rm II})-\delta\boldsymbol{H}^{\rm II*}\cdot(\boldsymbol{H}_0^{\rm I}-\boldsymbol{H}_0^{\rm II})\}\right)ds+cc}{\int_S\{|\boldsymbol{e}_{\rm in}|^2+Z_0^2|\boldsymbol{h}_{\rm in}|^2\}\,ds} \qquad (7)$$

where * indicates the complex conjugate and cc represents the complex conjugate of the previous term to this sign. Since the true fields must satisfy the following boundary conditions:

$$\boldsymbol{E}_0^{\rm I}=\boldsymbol{E}_0^{\rm II}$$
$$\boldsymbol{H}_0^{\rm I}=\boldsymbol{H}_0^{\rm II} \qquad (8)$$

on the discontinuity plane S, (7) reduces to

$$\delta F=0 \qquad (9)$$

for the correct fields. Therefore, the stationary problem associated with F is equivalent to the boundary value problem using the conventional boundary conditions (8). In the following analysis, the reflection, transmission, and radiation fields are determined in such a way that F becomes minimum. The electromagnetic fields thus obtained are the best approximate fields in the sense of the least square error. The condition for which F becomes minimum is given by

$$\frac{\partial F}{\partial X^*}=0 \qquad (10)$$

where X represents R, T, r_i, or t_i. Solving the simultaneous complex linear equations (10), the coefficients R, T, r_i, t_i, and, hence, the electromagnetic fields, can be obtained.

This is the outline of the least squares boundary residual technique which will be applied in the next section to treat the discontinuity problem in the dielectric slab waveguide.

To compare the results with those obtained by other techniques, two other methods used in the earlier paper are briefly summarized below for the convenience of reference.

B. *Mahmoud–Beal's Method* [6]

In Mahmoud–Beal's method, the conventional boundary conditions

$$E^{\rm I}=E^{\rm II} \qquad (11)$$
$$H^{\rm I}=H^{\rm II} \qquad (12)$$

are used instead of (5). Multiplying (11) and (12) by $\boldsymbol{h}_i^{\rm II*}$ or $\boldsymbol{H}_m^{\rm II*}$, $\boldsymbol{e}_i^{\rm II*}$ or $\boldsymbol{E}_m^{\rm II*}$, respectively, and integrating over the discontinuity plane S, the following simultaneous complex linear equations are derived:

$$\langle e_{in}, h_i^{II*}\rangle + \sum_{j=1}^{M} R_j \langle e_j^I, h_i^{II*}\rangle + \sum_{j=1}^{\infty} r_j \langle E_j^I, h_i^{II*}\rangle = \sum_{j=1}^{N} T_j \langle e_j^{II}, h_i^{II*}\rangle \qquad (i=1,\cdots,N)$$

$$\langle e_{in}, H_m^{II*}\rangle + \sum_{j=1}^{M} R_j \langle e_j^I, H_m^{II*}\rangle + \sum_{j=1}^{\infty} r_j \langle E_j^I, H_m^{II*}\rangle = \sum_{4=1}^{\infty} t_j \langle E_j^{II}, H_m^{II*}\rangle \qquad (m=1,\cdots,\infty)$$

$$\langle e_i^{II*}, h_{in}\rangle - \sum_{j=1}^{M} R_j \langle e_i^{II*}, h_j^I\rangle - \sum_{j=1}^{\infty} r_j \langle e_i^{II*}, H_j^I\rangle = \sum_{j=1}^{N} T_j \langle e_i^{II*}, h_j^{II}\rangle \qquad (i=1,\cdots,N)$$

$$\langle E_m^{II*}, h_{in}\rangle - \sum_{j=1}^{M} R_j \langle E_m^{II*}, h_j^I\rangle - \sum_{j=1}^{\infty} r_j \langle E_m^{II*}, H_j^I\rangle = \sum_{j=1}^{\infty} t_j \langle E_m^{II*}, H_j^{II}\rangle \qquad (m=1,\cdots,\infty) \quad (13)$$

where $\langle\ \rangle$ represents the inner product defined as

$$\langle E, H\rangle = \int_S \{E \times H\}\cdot i_z\, ds. \qquad (14)$$

Solving the foregoing equation (13), R_i, T_i, r_i, t_i, and, hence, the total electromagnetic fields can be obtained. The mean-square error of the obtained electromagnetic fields is larger than that associated with the least squares boundary residual method.

C. Marcuse's Method [7]

It is assumed that lines I and II support only one surface mode of propagation, and that the discontinuity is small so that the magnitudes of the reflection coefficients $|R|$ and $|R(\rho)|$ are very small in comparison with unity. It is assumed further that the electromagnetic fields of the modes in lines I and II are approximately orthogonal, namely,

$$\int_S \{e^I(\rho) \times h^{II*}\}\cdot i_z\, ds \ll 1 \qquad (15)$$

$$\frac{1}{2}\int_S \{e^I(\rho) \times h^{II*}(\rho')\}\cdot i_z\, ds \simeq \delta(\rho-\rho') \qquad (16)$$

where δ denotes the Dirac's delta function. Under these assumptions, equalizing the inner products of the fields in line I to those in line II in a similar manner as Mahmoud–Beal's methods, and neglecting the quadratic trivial terms, the reflection and the transmission coefficients of the guided modes and the radiation modes R, T and $R(\rho)$, $T(\rho)$ can be derived analytically as follows:

$$R = \frac{\int_S \{e^{II*} \times h_{in} - e_{in} \times h^{II*}\}\cdot i_z\, ds}{\int_S \{e^{II*} \times h_{in} + e_{in} \times h^{II*}\}\cdot i_z\, ds} \qquad (17)$$

$$T = \frac{2\int_S \{e^{II*} \times h_{in}\}\cdot i_z\, ds \int_S \{e_{in} \times h^{II*}\}\cdot i_z\, ds}{\int_S \{e^{II*} \times h_{in} + e_{in} \times h^{II*}\}\cdot i_z\, ds} \qquad (18)$$

$$R(\rho) = \frac{1}{2}\int_S \{e_{in} \times h^{II*}(\rho) - e^{II*}(\rho) \times h_{in}\}\cdot i_z\, ds \qquad (19)$$

$$T(\rho) = \frac{1}{2}\int_S \{e_{in} \times h^{II*}(\rho) + e^{II*}(\rho) \times h_{in}\}\cdot i_z\, ds. \qquad (20)$$

III. Transverse Displacement at a Junction of Two Dielectric Slab Waveguides

In this section, the transverse displacement of the dielectric slab waveguides as shown in Fig. 2 is analyzed as a typical and practically important example of discontinuity problem by using the three methods stated in the previous section. The numerical results obtained by the least squares boundary residual method are compared with those obtained by two other methods, and the features of each method are pointed out.

It is assumed that the incident wave is the TE dominant mode, and that only the lowest TE mode can propagate along two dielectric slab waveguides. The relative refractive index n of the guide is assumed to be 1.6, and the frequency is normalized in such a way as $k_0 d = 1.0$ where $2d$ is the thickness of the slab waveguide as shown in Fig. 2. Physically, for most of the radiation modes, the propagation constants ρ in the x-direction are very close to that of the incident mode. In other words, only a few radiation modes have large values of ρ. Since the Gauss–Laguerre functions decrease exponentially as ρ increases, they are suitable to use for the expansions of $R(\rho)$ and $T(\rho)$. The first three terms of the Gauss–Laguerre functions are

$$f_0(\rho) = \sqrt{\xi d}\ \exp\left(-\frac{\xi\rho d}{2}\right)$$

$$f_1(\rho) = \sqrt{\xi d}\ \{1-(\xi\rho d)\}\ \exp\left(-\frac{\xi\rho d}{2}\right)$$

$$f_2(\rho) = \sqrt{\xi d}\ \left\{1-2(\xi\rho d)+\frac{(\xi\rho d)^2}{2}\right\}\ \exp\left(-\frac{\xi\rho d}{2}\right)$$

$$\vdots \qquad (21)$$

and the orthogonality relations are

$$\int_0^\infty f_i(\rho) f_j(\rho)\, d\rho = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \qquad (22)$$

where ξ represents the concentration parameter which determines the shape of the spatial spectrum of the radiation modes.

Fig. 3 shows F given by (5) and the total power P_{total} obtained by the least squares boundary residual method versus ξ in (21) with the number of expansion terms as a parameter. P_{total} is the sum of the powers of reflection, transmission, and radiation waves which must be equal to 1 provided that the results obtained are correct. We can see from Fig. 3 that there exist particular values of ξ for which F becomes minimum. This means that, in the least squares boundary residual method, the optimum ap-

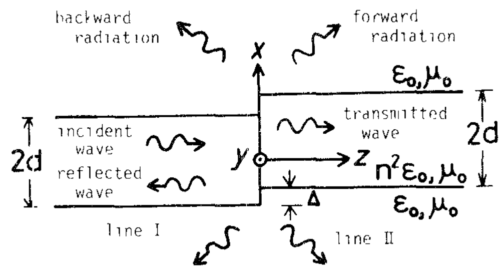

Fig. 2. Transverse displacement at the junction of two dielectric slab waveguides.

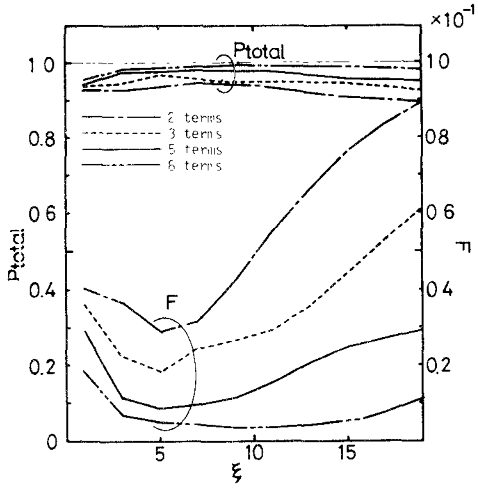

Fig. 3. Mean-square error F defined by (5) and the total power P_{total} evaluated by the least squares boundary residual method with the number of expansion terms as a parameter ($\Delta/2d=0.3$).

proximation can be obtained by choosing the value of ξ appropriately for each number of expansion terms.

Fig. 4 shows P_{total} evaluated by Mahmoud–Beal's method versus ξ. In this method, it seems to be reasonable to use the specific value of ξ as an adequate one for which P_{total} becomes unity. However, as we can see from Fig. 4, P_{total} becomes unity for two different values of ξ in the cases of two- and three-term expansions. Different values of ξ gives entirely different spatial spectra of the radiation power. Therefore, in Mahmoud–Beal's method, there is no reasonable criterion to find the suitable value of ξ which gives the best approximation for each number of expansion terms.

Fig. 5 shows the transmission power P_t, the radiation power P_{rad}, and the total power P_{total} evaluated by three different methods versus number of expansion terms. P_t and P_{rad} evaluated by the least squares boundary residual method converge to certain values, and the total power P_{total} approaches unity as the number of expansion terms is increased. The transmission power P_t and the radiation power P_{rad} calculated by Mahmoud–Beal's method are quite different from those obtained by the other two methods.

Fig. 6 shows the spatial spectrum of the forward radiation power calculated by the least squares boundary residual method with the number of expansion terms as a parameter. The shape of the spatial spectrum of the radiation power converges to a certain shape as the number of expansion terms is increased. The result obtained by Marcuse's method is also shown (by dotted line) in Fig. 6. As shown in Fig. 6, the spatial spectra of the radiation power evaluated by both methods become maximum at almost the same value of ρd (except $\rho d=1.0$). This means that both methods yield almost the same direction at

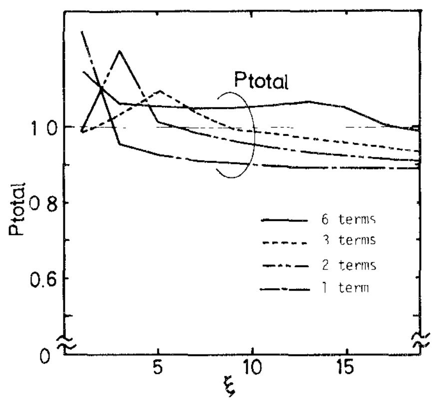

Fig. 4. Total power P_{total} evaluated by Mahmoud–Beal's method versus ξ ($\Delta/2d=0.3$).

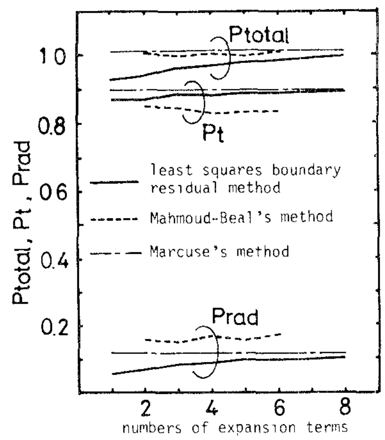

Fig. 5. Transmitted power P_t, radiated power P_{rad}, and total power P_{total} evaluated by three different methods versus number of expansion terms ($\Delta/2d=0.3$).

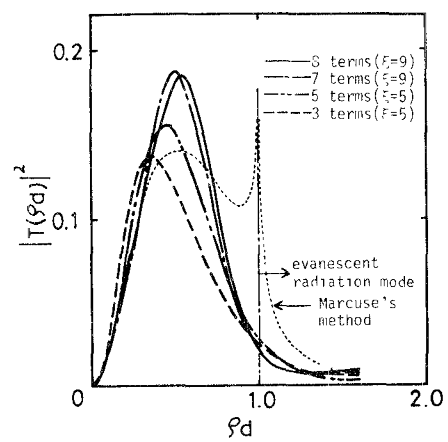

Fig. 6. Spatial spectra of the forwardly radiated power calculated by the least squares boundary residual method with the number of expansion terms as a parameter.

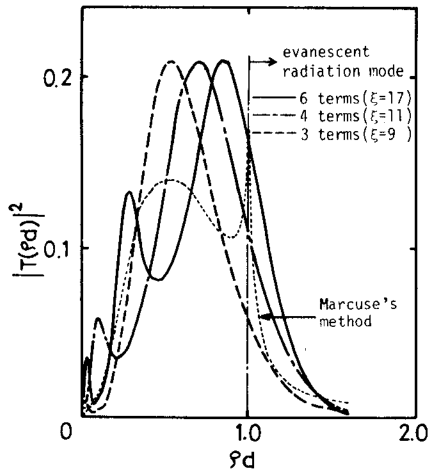

Fig. 7. Spatial spectra of the forwardly radiated power calculated by Mahmoud–Beal's method with the number of expansion terms as a parameter.

which the radiation power becomes maximum. However, the spatial spectrum obtained by Marcuse's method is not correct at the vicinity of $\rho d=1.0$. The reason for this is that the assumption $|R(\rho)|\ll 1$ in Marcuse's method no longer holds there. In other words, the spatial spectrum of the radiation power evaluated by Marcuse's method is not valid at the vicinity of $\rho d=1.0$. As shown in Fig. 5, the radiation power calculated by Marcuse's method is large in comparison with that obtained by the least squares boundary residual method because of the contribution of the integration of this unreasonable spatial spectrum of the radiation in the vicinity of $\rho d=1.0$. This, in turn, gives rise to the result that the total power P_{total} evaluated by Marcuse's method is much closer to unity in comparison with that obtained by the least squares boundary residual method, but it does not necessarily mean that Marcuse's method gives a better approximation because of the reasons stated above.

Fig. 7 shows the spatial spectrum of the forward radiation power calculated by Mahmoud–Beal's method with the number of expansion terms as a parameter. The result obtained by Marcuse's method is also shown (by dotted line) in the same figure. The spectrum is very different from those calculated by the least squares boundary residual method (Fig. 6) and Marcuse's method (dotted line). The rate of convergence in Mahmoud–Beal's method is poor in comparison with that in the least squares boundary residual method.

Fig. 8 shows the reflection, transmission, and radiation powers P_r, P_t, and P_{rad}, respectively, evaluated by the least squares boundary residual method versus the magnitude of the transverse displacement $\Delta/2d$. We can see from Fig. 8 that, in the case of the discontinuity of transverse displacement, the reflected power is much smaller than the radiated power. If the radiation modes are ignored, the transmission and reflection power can be obtained by very simple calculations. The results are shown in Fig. 8 by dashed lines.

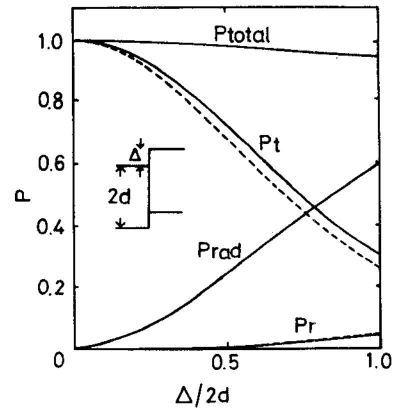

Fig. 8. Transmitted power P_t, reflected power P_r, radiated power P_{rad}, and total power P_{total} calculated by the least squares boundary residual method with eight expansion terms. Dashed lines (----) show P_t and P_r calculated simply by neglecting the radiation.

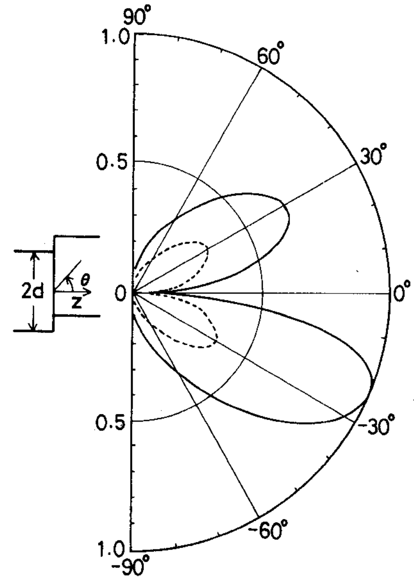

Fig. 9. Radiation patterns of electric fields. $\theta=0°$ is in z-direction. —— $\Delta/2d=1.0$, ---- $\Delta/2d=0.3$.

Fig. 9 shows the radiation patterns calculated by the steepest decent method using the spatial spectrum of the forward radiation power. $\theta=0°$ coincides with the z-direction.

IV. Conclusions

The least squares boundary residual method to treat the discontinuities in dielectric waveguides is presented. As an example of application, the reflection, transmission, and radiation waves produced by the transverse displacement at the junction of two single-mode dielectric slab waveguides are calculated. The results obtained are compared with those calculated by Mahmoud–Beal's method and Marcuse's method. The features of the proposed method are as follows. The rate of convergence of the numerical calculations of this method is relatively fast in comparison with Mahmoud–Beal's method. In comparing Marcuse's method, the present method can yield the reflected waves and is applicable even to large discontinuities. The present method is especially powerful to analyze the radiation characteristics due to the discontinuity. If we can ignore the radiation wave, the transmission and the reflection powers can be obtained very easily by the least squares boundary residual method.

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References

[1] D. L. Bisbee, "Measurements of loss due to offsets and end separations of optical fibers," *Bell Syst. Tech. J.*, vol. 50, no. 10, p. 3159, Dec. 1971.

[2] L. G Cohen, "Power coupling from GaAs injection lasers into optical fibers," *Bell Syst. Tech. J.*, vol. 51, no. 3, p. 573, Mar. 1972.

[3] J. S. Cook, W. L. Mammel, and R. J. Grow, "Effect of misalignments on coupling efficiency of single-mode optical fiber butt joints," *Bell Syst. Tech. J.*, vol. 52, no. 8, p. 1439, Oct. 1973.

[4] T. Sekizawa, "Components for optical-fiber transmission," *J. Inst. Electron. Commun. Eng. Japan*, vol. 59, no. 7, p. 728, July 1976.

[5] T. C. Chu and A. R. McCormick, "Measurements of loss due to offset, end separation, and angular misalignment in graded index fibers excited by an incoherent source," *Bell Syst. Tech. J.*, vol. 57, no. 3, p. 595, Mar. 1978.

[6] S. F. Mahmoud and J. C. Beal, "Scattering of surface waves at a dielectric discontinuity on a planar waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, p. 193, Feb. 1975.

[7] D. Marcuse, "Radiation losses of tapered dielectric slab waveguides," *Bell Syst. Tech. J.*, vol. 49, no. 2, p. 273, Feb. 1970.

[8] B. Rulf, "Discontinuity radiation in surface waveguides," *J. Opt. Soc. Amer.*, vol. 65, no. 11, p. 1248, Nov. 1975.

[9] G. H. Brooke and M. M. Z. Kharadly, "Step discontinuities on dielectric waveguides," *Electron. Lett.*, vol. 12, no. 18, p. 473, Sept. 1976.

[10] D. Gloge, "Offset and tilt loss in optical fiber splices," *Bell Syst. Tech. J.*, vol. 55, no. 7, p. 905, Sept. 1976.

[11] C. M. Miller, "Transmission vs transverse offset for parabolic-profile fiber splices with unequal core diameters," *Bell Syst. Tech. J.*, vol. 55, no. 7, p. 917, Sept. 1976.

[12] J. B. Davies, "A least-squares boundary residual method for the numerical solution of scattering problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, p. 99, Feb. 1973.

[13] H. Oraizi and J. Perini, "A numerical method for the solution of the junction of cylindrical waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, p. 640, Oct. 1973.

Computer-Aided Design of *H*-Plane Waveguide Junctions with Full-Height Ferrites of Arbitrary Shape

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***Abstract*—A method for solving the problem of *H*-plane waveguide junctions with a full-height ferrite post of arbitrary shape is proposed. The junctions are allowed to have arbitrary cross section and arbitrary number of ports. The method is based on the integral equations derived from the reciprocity theorems in both the ferrite region and the air region ranging from the reference planes of connecting waveguides to the inside of the junction.**

For comparison with the previously published experimental and theoretical results, Y junctions with a circular ferrite post are first treated. Excellent agreement has been found between the experimental data and the numerical results obtained by the present method.

The performance of a Y-junction circulator with a triangular ferrite post having rounded angles is next investigated. Both the ferrite geometry and the internal dc magnetic field are examined in detail. For this geometry the calculated 20-dB bandwidth has been found to become greater as the cross section of the ferrite approaches a regular triangle from a circle.

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I. Introduction

A VERSATILE waveguide junction which is now in wide use is the ferrite junction circulator. The waveguide Y-junction circulator was first proposed by Chait and Curry [1]. Davies [2] presented the theoretical treatment for a symmetrical waveguide junction circulator with a circular ferrite post, including a detailed field analysis inside the junction. This method was extended to junctions with coaxial composite ferrite posts which produced much larger bandwidths [3]–[5]. In these analyses, only the dominant-mode fields in the waveguides were approximately matched to a summation of mode fields within the junction. This neglect of the higher modes is the reason for the discrepancy between the numerical results and the experimental measurements. Later, this defect was improved by adding the higher modes [6] and by using the point-matching technique, and at the same

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